An Optimized Anisotropic PML for the Analysis of Microwave Circuits

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***Abstract*— The anisotropic perfectly matched layer (PML) is implemented in the finite-element method (FEM) to evaluate the S-parameters of microwave integrated circuits (MIC's). The PML region, which terminates the mesh over a range of frequencies, may exhibit either a uniform or nonuniform conductivity profile $\sigma(\rho)$. The performance of the PML is strongly dependent on the choice of $\sigma(\rho)$ as well as the mesh density inside the absorber. This observation is demonstrated numerically using a two-dimensional (2-D) finite-element analysis. The anisotropic PML is subsequently used in modeling three-dimensional (3-D) microwave integrated circuits. The accuracy and overall performance of the absorber is evaluated by computing the S-parameters of a low-pass filter.**

***Index Terms*—Finite-element method, perfectly matched layer.**

I. Introduction

SINCE the introduction of the perfectly matched layer (PML) [1], and later the anisotropic PML [2], many efforts have concentrated on the accurate implementation of such an absorber in finite-element method (FEM). Although obtained results demonstrate the potential of the anisotropic PML in computational electromagnetics [3], additional effort should be directed toward design and optimization of PML's. For example, design variables such as the conductivity profile, PML depth, and mesh size must be carefully chosen to improve the accuracy of the solution.

This letter presents an extensive investigation on the performance of the anisotropic PML absorber in the context of the FEM. Various parametric studies on the accuracy of the PML are systematically carried out for different conductivity profiles. Two distinct choices are considered in this study: a uniform and a nonuniform conductivity profile. Either choice of profile may result in similar results provided that the discretization error does not become a factor. Design guidelines and suggestions concerning proper use of the anisotropic PML are provided in the following sections. These conclusions are based on a two-dimensional (2-D) analysis of a parallel-plate waveguide terminated with a conductor-backed PML region.

Once evaluating the overall performance of the anisotropic PML in 2-D guided structures, the concept is applied to analyze the S-parameters of a low-pass microwave filter. The anisotropic PML is used to terminate ports and open sidewalls. A uniform conductivity profile is chosen to characterize the PML medium. The results are compared with finite-difference time-domain (FDTD) and FEM based on traditional dispersive absorbing boundary conditions (ABC's).

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II. Theory

The uniaxial structure of the permittivity and permeability tensors characterizing the anisotropic PML is given for the case of a z-directed traveling wave by [2]

$$\frac{[\epsilon]_z}{\epsilon_\circ \epsilon_r} = \frac{[\mu]_z}{\mu_\circ \mu_r} = \Lambda_z = \left\{\kappa, \kappa, \frac{1}{\kappa}\right\} \tag{1}$$

where

$$\kappa = 1 - j\frac{\sigma(\rho)}{\omega \epsilon_\circ}. \tag{2}$$

For a wave traveling in the x-direction, the uniaxial tensors should be rotated by 90° about y-axis. Similarly, for a traveling wave along the xy-direction, the PML permittivity and permeability tensors should be proportional to the product of Λ_x and Λ_y [4]. All remaining combinations are formed in a similar manner. Note also that the rate with which the incident field is attenuated inside the PML region is related to $\sigma(\rho)$. In numerous results presented by Berenger [1], it was clearly pointed out that the numerical reflections from the PML interface can be significantly reduced by carefully selecting the conductivity profile. Usually, numerical reflections occur when the transition from one medium to another becomes more abrupt. Thus a judicious choice for $\sigma(\rho)$ is [1]

$$\sigma(\rho) = \sigma_{\max}\left(\frac{\rho - \rho_\circ}{d}\right)^m \tag{3}$$

where $\sigma_{\max}$ is the maximum conductivity value of the anisotropic medium, d is the entire depth of the PML region, m is the order of the spatial polynomial, and $\rho_\circ$ is the position of the PML interface in the direction of propagation. The spatial decay of the field inside the PML region is mainly controlled by the actual value of $\sigma_{\max}$. A good choice for $\sigma_{\max}$ is given by

$$\sigma_{\max} = \frac{(m+1)\epsilon_\circ v}{2d} \ln\left(\frac{1}{R}\right) \tag{4}$$

where R is defined as the theoretical reflection coefficient at normal incidence and v is the velocity of the traveling wave just before it reaches the PML interface. Setting the reflection coefficient to a desired value, e.g., $1.0e{-}4$, a good estimate for $\sigma_{\max}$ can be obtained. Equation (4) does not take into account the discretization error.

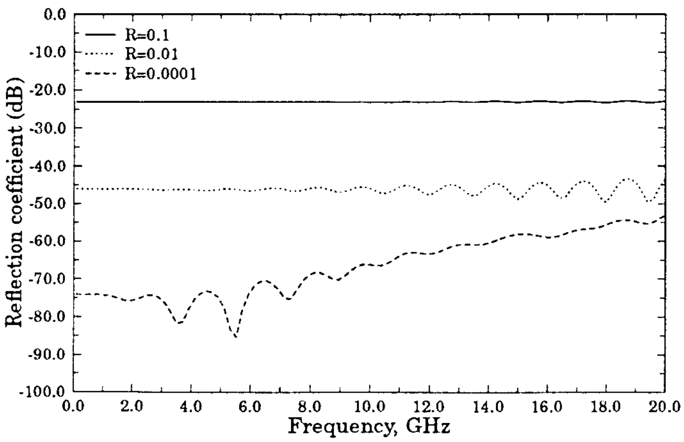

Fig. 1. Reflection coefficient versus frequency for various values of R ($N = 10$, $d = 20$ mm, $m = 2$).

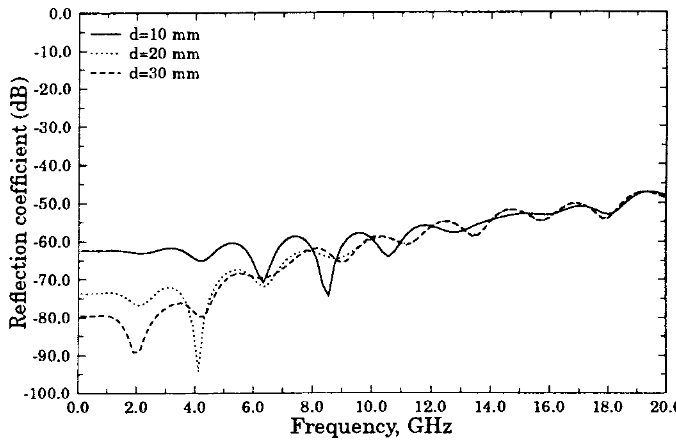

Fig. 2. Reflection coefficient versus frequency for various values of d ($N = 10$, $R = 1.0e{-}4$, $m = 2$).

The continuous distribution of $\sigma(\rho)$ inside the PML medium is staircased based on the number of layers involved. A layer is defined as a rectangular region of uniform conductivity which, depending on the mesh size, might accommodate more than one element in the ρ-direction. Each layer exhibits a distinct conductivity value. An alternative choice is the use of a uniform instead of a nonuniform conductivity profile. In such a case, the spatial variation of the material conductivity $\sigma(\rho)$ inside the PML is set to a constant; i.e., $m = 0 \Rightarrow \sigma(\rho) = \sigma_{\max}$. Such a choice of σ though results in an abrupt material discontinuity thereby creating numerical reflections due to insufficient discretization near the interface. A finer discretization inside the PML approximates better the rapid variation of the incident field; thus, smaller reflections occur.

III. Results

The accuracy of the anisotropic PML absorber was first investigated in the case of a parallel-plate waveguide excited with the TEM mode. The parallel-plate waveguide is air-filled and terminated with a nonuniform PML region backed with a perfect electric conductor. The depth of the PML is 20 mm and the number of layers (N) is 10. The average mesh size is 1 mm. The order of the spatial polynomial, shown in (3), is 2 (quadratic profile). The accuracy of a nonuniform conductivity profile is investigated for various values of R which, as indicated by (4), directly relates to $\sigma_{\max}$. The resulting reflection coefficient as a function of frequency is depicted in Fig. 1. It is interesting to observe that the obtained numerical reflection coefficient reduces substantially when the desired value of R decreases from 0.1 to 0.0001. Note also that for larger values of R, the reflection coefficient remains constant versus frequency, which is expected according to the theory supporting the PML concept. However, for smaller values of R, the reflection coefficient is dominated by the discretization error, which increases with increasing frequency.

The same air-filled parallel-plate waveguide was chosen to investigate the effect of changing the depth of the PML region while maintaining all remaining parameters constant. The reflection coefficient versus frequency for three different values of d is shown in Fig. 2. This figure illustrates that by increasing the depth of the PML region, the reflection coefficient at the lower frequency range becomes smaller. Specifically, increasing the PML depth by three times the original value, the reflection coefficient at the lower end of the frequency range is reduced by almost 20 dB. The reason relates to the fact that by increasing d, $\sigma_{\max}$ becomes smaller; therefore, the field inside the PML region decays more gradually, which obviously means less discretization error. The smaller the PML depth, the denser the grid should be in order to achieve the desirable accuracy. Based on the 2-D analysis, it was found that by choosing the mesh size h inside the PML region to be approximately equal to $d/10$ and setting $N \geq 2$, the numerical reflection coefficient from the PML termination is on the order of -60 dB.

It was also found that by increasing N larger than 2 does not have a significant effect on the accuracy of the PML, provided that d and mesh density remain unchanged [5]. However, using a single-layer PML (uniform conductivity profile), the solution accuracy deteriorates substantially. The reason is that σ is now set to $\sigma_{\max}$, thus forcing the wave to attenuate quite rapidly inside the PML. In such a case, the discretization error, which is caused by poor approximation of the field variation near the PML interface, is substantially larger than the discretization error observed when using a nonuniform conductivity profile. This type of error can be significantly reduced by using a finer mesh inside the PML region. Such improvement is illustrated numerically in Fig. 3 for a parallel-plate waveguide that is terminated at the output port using a uniform profile PML medium of depth equal to 20 mm. This figure provides a comparison of the numerical reflection coefficient versus δ ($= \ln(1/R)$) for three different mesh densities: mesh #1 has an average cell size of 0.5 mm; mesh #2 has an average cell size of 1.0 mm; and mesh #3 has an average cell size of 2.0 mm. The operating frequency is set to 100 MHz. Although only a uniform conductivity profile, the numerical reflection coefficient closely follows the theoretical reflection coefficient, obtained by rearranging (4), up to a level of -65 dB where the discretization error begins to dominate. To reach even lower than -65 dB, let us say -75 dB, which is the error level

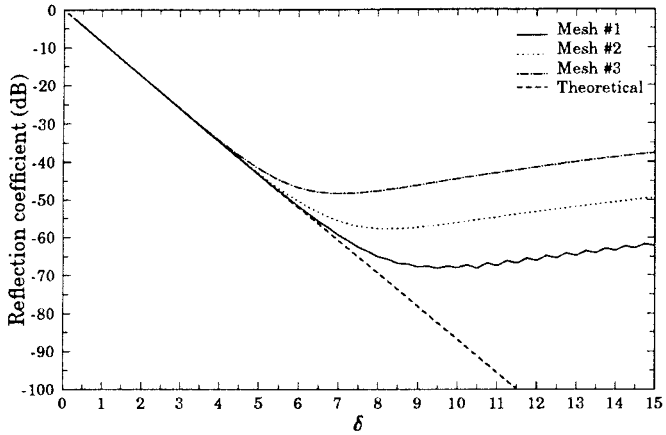

Fig. 3. The effect of discretization error in optimizing a single-layer PML medium ($\delta = \ln(1/R)$).

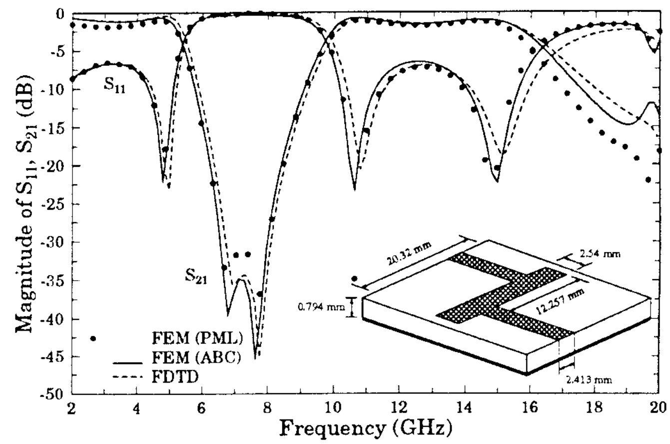

Fig. 4. S-parameters of a low-pass filter printed on a RT/Duroid substrate with $\epsilon_r = 2.2$.

obtained previously using a nonuniform conductivity profile with mesh size of just 1 mm, the triangular mesh inside the PML should be additionally refined. This has been proven to be computationally expensive for three-dimensional (3-D) problems.

Based on the 2-D analysis, it is clear that the use of a nonuniform conductivity profile results in a higher degree of accuracy compared to a uniform conductivity profile, unless a very fine mesh is used for the second case. Unfortunately, the nonuniform conductivity profile introduces geometrical complexities due to the unstructured nature of the mesh. In other words, the PML region has to be partitioned into N rectangular segments before meshing. In addition, each segment has to be assigned with a distinct material number. To avoid such complexities, a uniform conductivity profile is implemented for the analysis of microwave circuits. In this case, the circuit is surrounded only by one layer of absorbing material.

A vector finite-element code was implemented to compute the S-parameters of a microwave low-pass filter. This circuit was originally analyzed by Sheen *et al.* [6] using the FDTD method. The depth of the single-layer PML region is only 3 mm and the distance to the nearest discontinuity is also 3 mm. The geometry was discretized using linear tetrahedral elements. The average size of the element used is approximately 1 mm, whereas the total number of elements is 27 854. Note that only three elements exist (along the normal direction) inside the PML region; this is not satisfactory. At least ten elements are needed to guarantee accuracy close to -50 dB. A 2-D full-wave finite-element analysis is used to obtain the fundamental mode distribution at the input port, which is then used as the field excitation for the 3-D circuit. The magnitude of S_{11} and S_{21} is illustrated in Fig. 4. These numerical results are compared with data obtained using both the FDTD method and the FEM based on dispersive ABC's. The comparison among the three data sets illustrates good agreement. At the higher frequencies, however, the discrepancy between the two data sets becomes more visible, which seems to be attributed to an increasing discretization error in both methods.

IV. Conclusion

A finite-element analysis was used to investigate the performance of perfectly matched layers in discretized domains. Assuming the mesh density remains the same, a nonuniform conductivity profile, staircased with at least two layers, provides better results than a uniform conductivity profile. A refined mesh inside the PML always improves the solution accuracy. An optimized anisotropic PML was found not only simple to implement in FEM but also accurate in predicting scattering parameters of microwave circuits. A serious disadvantage of this concept, however, is the resulting slow convergence for iterative solvers. At present, the use of an ABC in FEM is computationally more efficient than PML, but not necessarily more accurate. Future developments on the subject suggest that the PML will most likely be the preferred approach to truncating the finite-element domain.

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